Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics
# Higher level
# Paper 3 – calculus

Tuesday 10 November 2020 (afternoon)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.



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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 7]

Use l'Hôpital's rule to find

$$\lim_{x \to 1} \frac{\cos(x^2 - 1) - 1}{e^{x-1} - x}.$$

**2.** [Maximum mark: 10]

Consider the series $\sum_{n=1}^{\infty} \frac{(5-3x)^n}{n}$.

(a) Show that the series is convergent for $\frac{4}{3} < x < 2$. [5]

(b) Find the interval of convergence of the series. [5]

**3.** [Maximum mark: 18]

The curve $y = f(x)$ has a gradient function given by

$$\frac{dy}{dx} = x - y.$$

The curve passes through the point $(1, 1)$.

(a) (i) On the same set of axes, sketch and label isoclines for $\frac{dy}{dx} = -1$, $0$ and $1$, and clearly indicate the value of each $y$-intercept.

(ii) Hence or otherwise, explain why the point $(1, 1)$ is a local minimum. [6]

(b) Find the solution of the differential equation $\frac{dy}{dx} = x - y$, which passes through the point $(1, 1)$. Give your answer in the form $y = f(x)$. [8]

(c) (i) Explain why the graph of $y = f(x)$ does not intersect the isocline $\frac{dy}{dx} = 1$.

(ii) Sketch the graph of $y = f(x)$ on the same set of axes as part (a)(i). [4]

**4.** [Maximum mark: 15]

The function $f$ is defined by $f(x) = \ln(1 + x^2)$ where $-1 < x < 1$.

(a) (i) Use the Maclaurin series for $\ln(1 + x)$ to write down the first three non-zero terms of the Maclaurin series for $f(x)$.

(ii) Hence find the first three non-zero terms of the Maclaurin series for $\frac{x}{1+x^2}$. [6]

(b) Use your answer to part (a)(i) to write down an estimate for $f(0.4)$. [1]

The seventh derivative of $f$ is given by $f^{(7)}(x) = \frac{1440x(x^6 - 21x^4 + 35x^2 - 7)}{(1+x^2)^7}$.

(c) (i) Use the Lagrange form of the error term to find an upper bound for the absolute value of the error in calculating $f(0.4)$, using the first three non-zero terms of the Maclaurin series for $f(x)$.

(ii) With reference to the Lagrange form of the error term, explain whether your answer to part (b) is an overestimate or an underestimate for $f(0.4)$. [8]